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Classical and Simple Adaptive Control for Nonminimum Phase Autopilot Design

Itzhak Barkana*
Kulicke and Soffa Industries, Inc., Willow Grove, Pennsylvania 19090

A recent publication uses a difficult design example to show that fuzzy logic might have advantages when compared with classical compensators. Although in this particular case the application was shown to be successful, convergence of the fuzzy-logic algorithm, as well as other time-varying controllers, cannot not be guaranteed unless some preliminary conditions are satisfied. It will be shown that further exploitation of the classical design can improve robust performance. This result is then used to create sufficient conditions that guarantee convergence with time-varying controllers, and it is then shown that simple adaptive control methods can further improve performance and maintain it in changing environments.

I. Introduction

A RECENT publication[1] has presented successful applications of fuzzy-logic control design in a nonminimum phase autopilot with uncertainty of parameters. The authors use this difficult design case to show that fuzzy logic has advantages when compared with a classical compensator or with the ubiquitous proportional–integral–derivative (PID) design when uncertainty is concerned. Although this particular fuzzy-logic application was successful, it is well known that convergence with nonstationary controllers, including adaptive and fuzzy-logic algorithms, is not inherently guaranteed. This paper intends to show that further exploitation of the basic knowledge of the plant and the uncertainty can be used to improve the performance of a classical control design and also to create sufficient conditions that guarantee convergence of time-varying controllers. The results are presented here in connection with simple adaptive control that is shown to achieve improved performance along with the guarantee of stability.

Successful implementations of simple direct adaptive control techniques in various domains of application have been presented over the past two decades in the technical literature. This simple-adaptive-control (SAC) methodology has been introduced by Sobel et al.[2] and further developed by Barkana et al.[3] and Barkana and Kaufman.[4,5] These techniques have also been extended by Wenn and Balas[6] and Balas[7] to infinite-dimensional systems. Those successful applications of low-order adaptive controllers to large-scale examples have led to successful implementations of SAC in such diverse applications as flexible structures,[8–15] flight control,[16,17] power systems,[18] robotics,[19] motor control,[20,21] drug infusion,[22,23] and other.[24]

SAC methodology had started with an apparently restricted range of applications because it seemed to be feasible only for step input commands and required the controlled plant to be almost strictly passive (ASP) and its transfer function almost strictly positive real (ASPR). A linear-time-invariant (LTI) plant is called ASP and its transfer function ASPR[5] if there exists an output feedback gain (fictitious, unknown, and not needed for implementation) that can stabilize the plant and also make it strictly positive real (SPR). The SPR properties are the principal prior conditions needed for the proofs of asymptotic stability of nonstationary, nonlinear, and adaptive systems. When SAC was first developed, it was not clear what classes of plants satisfied these conditions. However, it has been shown that plants with minimum-phase transfer functions of relative degree 1 in the SISO case,[25,26] or of relative degree m in the m-input–m-output multivariable case,[27–32] are ASPR.

In spite of some specific exceptions, most realistic plants might not be ASPR. However, it has been subsequently shown how various forms of parallel feedforward configurations (PFC) can be used to satisfy the ASPR condition, thus extending SAC applicability to almost all practical applications.[5,25,33–38] A structural decomposition method has also been proposed that allows satisfaction of the ASPR condition without the use of parallel feedforward.[39] Kaufman et al.[40] summarize various developments concerning the theory and applications of SAC.

Simple-adaptive-control techniques can make the performance of the augmented system arbitrarily close to the ideal desired performance represented by an ideal model. It must be emphasized that although now the control variable is the augmented output the desired control variable remains the output of the original plant. This is a potential weakness of the PFC methodology, and it is very important, therefore, to guarantee that the PFC addition is sufficiently "small" so the behavior of the actual plant output is very similar to the ideal behavior of the augmented system. The ASPR condition that is now satisfied by the augmented system also guarantees convergence with various identification algorithms,[41] thus allowing the level of plant knowledge used in the design to be upgraded.

It was also recently shown[42] that the adaptive control gains perform a steepest descent error minimization in the SAC methodology. It was also shown that the adaptive controllers can achieve higher performance than the constant controllers because the conditions that guarantee asymptotically perfect tracking with the simple adaptive control system are much less demanding than in the LTI case. In particular, because the adaptive controller fits the control parameters to the specific problem at task asymptotic adaptive tracking is possible even when the general linear-time-invariant tracking solution does not exist.[42]

The present paper shows how to use the available prior knowledge about the stabilizability properties and about the uncertainty related to the plant, in order to guarantee stability with nonstationary controllers. Simple-adaptive-control techniques are then called and used to obtain the desired performance in changing environments.

II. Formulation of the Problem

The first part of the paper shows how to use the basic knowledge of the autopilot, including the uncertainty in parameters, and the preliminary control design used in Cohen and Bossert[1] to satisfy the ASPR conditions and to allow implementation of SAC. It is shown

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*K&S Fellow, 2101 Blair Mill Road; ibarkana@kns.com. Senior Member AIAA.

that the augmented ASPR system tracks the ideal model almost perfectly. However, if the additional output term introduced by the parallel feedforward configuration is not small, the behavior of the actual autopilot output can be far from desired. However, a second look at the plant will show that the preliminary control design can be improved such that satisfaction of the ASPR conditions is possible with small PFC and thus can guarantee both stability with adaptive controllers and good and consistent performance at the actual plant output and along the entire range of uncertainty.

The SAC methodology requires the plant, which could be of very large dimension, to track a model, without requiring the model to be of same order as the plant. Actually, the model is a representation of the plant only as far as its output represents the desired output behavior of the plant. It could be a first-order low-pass filter incorporating only the desired plant time constant or a linear system just large enough to generate a general desired command. Thus, the so-called model is actually a command generator, and therefore this technique has also been called command generator tracker (CGT).[43]

General adaptive control methodologies can assume that the plant parameters are totally unknown. Therefore, indirect adaptive-control methods use identification of all plant parameters along with the control of the plant. Direct adaptive-control methods compute directly the controller parameters, but the controller must be of the same order as the plant. Both methods, therefore, require knowledge of the order of the plant or the assumption that a nominal model of the plant is dominant over the unmodeled uncertainty.

The SAC methodology instead assumes that some prior knowledge on the stability or the stabilizability properties of the plant to be controlled is available. Although the order or even the exact structure of the plant might not be necessarily known, some frequency-response plots from experimental tests or some basic knowledge on the existence of some stabilizing controller (not necessarily "good") could be available. This basic knowledge can be used to test the ASPR properties of the plant or to build the proper PFC that can make the use of SAC safe and robust. Therefore, SAC is by no means an attempt to be the general solution for the general problem. On the contrary, the SAC methodology encourages using any basic knowledge and any useful ideas from the designer's own engineering experience, such as basic control design of the nominal plant or robust control design of the plant with uncertainty, before the use of SAC. SAC can then be used in an attempt to further improve performance and to maintain the desired performance over the entire range of uncertainty.

III. Ideal Control and Ideal Trajectories

Let the plant be a linear, time-invariant finite-dimensional system described as

$$\dot{x}_p = A_p x_p + B_p u_p \tag{1}$$

$$y_p = C_p x_p \tag{2}$$

The plant is required to asymptotically track the output of the model:

$$\dot{x}_m = A_m x_m + B_m u_m \tag{3}$$

$$y_m = C_m x_m \tag{4}$$

The plant is considered to be of (possibly) very large order n_p, with m inputs and m outputs, whereas the model can be of any order n_m, even very low, just sufficiently large to create the desired command for the plant. The input u_m is a step input function [in the single-input/single-output (SISO) case] or a vector of step input functions of desired arbitrary magnitudes (in the multivariable case) that excite the model such that its output y_m is the desired command.[40]

For simplicity of presentation, we assume here that the command generator uses the step input u_m to generate the desired command y_m to the plant. If step input is not sufficient to generate the desired command, or if one wants to test the system with unknown input commands, the input u_m itself can be a generalized input signal.[40]

In the CGT methodology, one uses the available knowledge of the model and forms the following ideal control input to the plant:

$$u_p^* = \tilde{K}_e(y_m - y_p) + \tilde{K}_x x_m + \tilde{K}_u u_m \tag{5}$$

Here, $\tilde{K}_e$ is any stabilizing gain, while $\tilde{K}_x$ and $\tilde{K}_u$ are the ideal feedforward control gains. When the ideal control is such that perfect output tracking occurs,

$$e_y = y_m - y_p = 0 \tag{6}$$

we say that the plant moves along ideal trajectories that allow this perfect tracking. At that time

$$u_p^* = \tilde{K}_x x_m + \tilde{K}_u u_m \tag{7}$$

These ideal trajectories are given by a linear combination of similar form

$$x_p^* = S_{11} x_m + S_{12} u_m \tag{8}$$

These ideal trajectories must be bounded trajectories of the plant and must satisfy the plant equations (1) and (2). Substituting Eqs. (7) and (8) in the plant equation and comparing with the derivative of x_p^* in Eq. (8) gives the perfect tracking conditions[18,31]:

$$\begin{bmatrix} A_p & B_p \\ C_p & 0 \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ \tilde{K}_x & \tilde{K}_u \end{bmatrix} \begin{bmatrix} x_m \\ u_m \end{bmatrix} = \begin{bmatrix} S_{11} A_m & S_{11} B_m \\ C_m & 0 \end{bmatrix} \begin{bmatrix} x_m \\ u_m \end{bmatrix} \tag{9}$$

A fixed coefficient solution for the matrical equation (9) leads to the following matching conditions for LTI output tracking:

$$\begin{bmatrix} A_p & B_p \\ C_p & 0 \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ \tilde{K}_x & \tilde{K}_u \end{bmatrix} = \begin{bmatrix} S_{11} A_m & S_{11} B_m \\ C_m & 0 \end{bmatrix} \tag{10}$$

If solutions of Eq. (10) exist, then the adaptive algorithm should eventually converge to one of these ideal solutions. However, although satisfaction of Eq. (10) is convenient for the proofs of stability, it was shown[15] that the adaptive control gains do not necessarily converge to the values predicted by Eq. (10). In an attempt to solve this discrepancy, it was recently shown[42] that asymptotic tracking with adaptive control is guaranteed even if Eq. (9) is only satisfied at steady state, when both u_m and x_m are constant. This leads to the equation

$$\begin{bmatrix} A_p & B_p \\ C_p & 0 \end{bmatrix} \begin{bmatrix} S_{11} x_{\mathrm{mss}} + S_{12} u_m \\ \tilde{K}_x x_{\mathrm{mss}} + \tilde{K}_x u_m \end{bmatrix} = \begin{bmatrix} 0 \\ C_m x_{\mathrm{mss}} \end{bmatrix} \tag{11}$$

Here, x_{mss} is the constant steady-state value of the model state. It was shown[42] that Eq. (11) always has solutions under the ASPR conditions. Therefore, the ASPR property is actually the only condition required to guarantee asymptotically perfect tracking with SAC.

IV. Simple Adaptive Controller

When one cannot assume full knowledge of the plant parameters, or when it is suspected that variable gains might be needed in a particular environment, an adaptive algorithm can be used to implement the control gains if stability and asymptotic tracking can be guaranteed. SAC methodology replaces the constant gains with time-varying adaptive gains:

$$u_p = K_e(t)[y_m - y_p] + K_x(t) x_m + K_u(t) u_m \tag{12}$$

The gains are generated by an adaptive algorithm that must maintain stability of the controlled system and bring the tracking error to zero. The plant tracks the model perfectly when it moves along the ideal trajectories x_p^*; otherwise, there is a state error e_x between the desired trajectory and the actual plant trajectory:

$$e_x = x_p^* - x_p \tag{13}$$

It is easy to see that

$$e_y = y_m - y_p = C_p x_p^* - C_p x_p = C_p e_x \tag{14}$$

One uses the measurable tracking error $e_y(t)$ to generate integral adaptive control gains $K_x(t)$ and $K_u(t)$, namely,

$$\dot{K}_{Ix}(t) = e_y x_m^T \Gamma_{Ix} \tag{15}$$

$$\dot{K}_{Iu}(t) = e_y u_m^T \Gamma_{Iu} \tag{16}$$

where Γ_{Ie} and Γ_{Ix} are constant matrical coefficients that control the rate of adaptation.

Although the control gains (15) and (16) have been initially developed from stability arguments,[2] it has been recently shown[42] that they perform a steepest descent minimization of the tracking errors. If the adaptive gains were to perform minimization of a function that depended only on $K_x(t)$ and $K_u(t)$, the algorithm (15) and (16) would be the entire procedure. However, the tracking error is the result of a dynamic plant behavior, and this plant could be even unstable. Therefore, an adaptive integral error gain $K_e(t)$, introduced to maintain stability of the system, was similarly devised to be

$$\dot{K}_{Ie}(t) = e_y e_y^T \Gamma_{Ie} \tag{17}$$

where Γ_{Iu} is a constant matrical coefficient.

Equations (15–17) can be written in a concise form by defining

$$r(t) = \begin{bmatrix} e_y \\ x_m \\ u_m \end{bmatrix} \tag{18}$$

$$K_I(t) = [K_{Ie}(t) \quad K_{Ix}(t) \quad K_{Iu}(t)] \tag{19}$$

With this notation we get

$$\dot{K}_I(t) = e_y r^T \Gamma_I \tag{20}$$

where Γ_I is the resulting matrical adaptation coefficient for the combined gain $K_I(t)$.

We sometimes add a proportional adaptive gain

$$K_p(t) = e_y r^T \Gamma_p \tag{21}$$

and use the total adaptive gain $K(t) = K_I(t) + K_p(t)$.

Although only the integral adaptive gain (20) is absolutely needed for stability and asymptotic tracking of the adaptive system, the examples and the proofs of stability[40] show that the proportional gain has the effect of increasing the rate of convergence of the system towards perfect tracking.

The simple adaptive control signal is

$$u_p = K(t) r(t) \tag{22}$$

The adaptive control system is composed of both the differential equations of the plant and of the adaptive gains:

$$\dot{x}_p = A_p x_p + B_p u_p = A_p x_p + B_p K(t) r \tag{23}$$

$$\dot{K}_I(t) = e_y r^T \Gamma_I \tag{24}$$

Subtracting the plant equation from the ideal trajectory gives, after some algebra, the differential equation of the state error:

$$\dot{e}_x = (A_p - B_p \tilde{K}_e C_p) e_x - B_p (K(t) - \tilde{K}) r(t) \tag{25}$$

V. Brief Review of Stability

The proof of stability must deal with both Eqs. (24) and (25) and must show that the errors converge and that the control gains are bounded. For the proof of stability, it is assumed that the plant is ASPR. In other words, the plant is stabilizable through some constant output feedback $\tilde{K}_e$, unknown and not needed for implementation. In addition, the fictitious closed-loop system with the system matrix

$$A_c = A_p - B_p \tilde{K}_e C_p \tag{26}$$

satisfies the Kalman–Yakubovich–Popov conditions

$$P A_c + A_c^T P = -Q \tag{27}$$

$$P B_p = C_p W \tag{28}$$

Here, P, Q, and W are positive-definite matrices.

It is well known that conditions (27) and (28) are equivalent to the SPR of the fictitious closed-loop system $\{A_c, B_p, C_p, 0\}$. Therefore, the original plant is called ASPR[5,25] because a constant output feedback is sufficient to make it strictly positive real. The reader might be accustomed to the common form of Eq. (28), where W is the identity matrix, implying that positive realness is restricted to those systems where the product $C_p B_p$ is positive-definite symmetric. However, it was recently shown[44] that this apparently limiting symmetry condition can be eliminated. A positive-definite symmetric matrix $W = S^T S$ exists, which makes the product $W C_p B_p$ positive-definite symmetric whenever $C_p B_p$ is positive definite but not symmetric.

One can chose a positive-definite quadratic Lyapunov function of the form

$$V(x) = e_x^T P e_x + \mathrm{trace}[S^T (K(t) - \tilde{K})^T \Gamma_I^T (K(t) - \tilde{K}) S] \tag{29}$$

The ASPR conditions (27) and (28) are needed because the derivative of the Lyapunov function is then

$$\dot{V}(x) = -e_x^T Q e_x \tag{30}$$

The Lyapunov function (29) is positive-definite quadratic in terms of all state variables of the dynamical system $\{e_x, K(t)\}$. The Lyapunov derivative (30), however, only includes the state error e_x and is therefore negative definite in the following error e_x and negative semidefinite in the state-gain space $\{e_x, K(t)\}$. The Lyapunov–LaSalle theory[40,45] then tells us that the system is stable, the errors vanish asymptotically,[40] and the gains end with some constant steady-state value.[42]

Although some systems, such as flexible structures with collocated sensors and actuators, could be inherently ASPR,[46,47] most systems are definitely not. For systems that are not ASPR, it has been shown[5,25,33–38] that parallel feedforward can be used to fulfill the required conditions.

The basic parallel feedforward idea is quite simple[25]: if a system $G(s)$ is known to be stabilizable thorough a controller $H(s)$, then the inverse $H^{-1}(s)$ used in parallel with the original plant makes the augmented system $G_a(s) = G(s) + H^{-1}(s)$ minimum phase. The original $G(s)$ could be both unstable and nonminimum phase. When the final relative degree is one (in SISO systems) or m (in m-input–m-output multivariable systems), the resulting augmented system is ASPR. Therefore, passivability has been shown to be dual to stabilizability. The poles of the augmented system $G_a(s)$ are not affected by the use of PFC and could be unstable, but $G_a(s)$ can now be used safely with SAC, and the stability of the system and the asymptotic tracking of the resulting adaptive control system are guaranteed.

We must also note, however, that the adaptive gain (17) would increase whenever the tracking error is not zero. Although the brief analysis following Eq. (30) implies that all adaptive gains converge to constant finite values under ideal conditions,[42] the gain $K_{Ie}(t)$ would continually increase in the presence of any noise, even at those noise levels that are negligible for any other practical purposes. Although an ASPR system remains stable with arbitrarily high gains, these gains might be too high for any practical (and numerical) purpose and might even diverge in time. This effect is not experienced by the control gains $K_{Ix}(t)$ and $K_{Iu}(t)$ that move up and down according to the specific situation. Therefore, we adopt Ioannou's idea[48] and add a sigma term (or forgetting factor) in Eq. (17) such that the error integral gain is

$$\dot{K}_{Ie}(t) = e_y e_y^T \Gamma_{Ie} - \sigma K_{Ie}(t) \tag{31}$$

With this adjustment, the error gain $K_{Ie}(t)$ increases whenever it is required to increase (because of large errors, etc.) and decreases when large gains are not needed anymore. The coefficient σ can be very small because its aim is only to prevent the error adaptive gain $K_{Ie}(t)$ from reaching excessively high values or diverging in time.

VI. Example

For illustration and for a better understanding of the problem, we select here a nonminimum phase autopilot for the altitude hold system of the Tower Trainer 60 unmanned aerial vehicle.[1] Although the plant is not unknown, its particular frequency response and the variation of its parameters, namely, the stability and control derivatives in various environments, make it a rather difficult system to design by any control methodology.

At nominal design conditions, the altitude (actually the change in altitude from a steady-state level flight condition) to elevator transfer function $h(s)/\delta_e(s)$ is

$$\frac{h(s)}{\delta_e(s)} = G_1(s) = \frac{-34.16s^3 - 144.4s^2 + 7047s + 557.2}{s^5 + 13.18s^4 + 95.93s^3 + 14.61s^2 + 31.94s} \tag{32}$$

The change in various conditions, so the plant transfer function at degraded conditions represented by a 50% reduction in the control parameters $C_{m\alpha}$ and C_{mq} is

$$\frac{h(s)}{\delta_e(s)} = G_2(s) = \frac{-34.16s^3 - 62.64s^2 + 8252s + 715.9}{s^5 + 10.79s^4 + 48.61s^3 + 7.852s^2 + 15.96s} \tag{33}$$

Figure 1 shows the block diagram of the adaptive control system including the plant, the actuator, the LTI controller, the parallel feedforward configuration, the reference model, and the adaptive controller.

The Bode plots for both plants (Fig. 2) shows that the dc gain is very high in both cases and that the frequency response must be appropriately adjusted. This is a good illustration for a practical control design. Although here we computed the data for the frequency-response plots from the transfer functions, in reality they could be obtained experimentally, from wind-tunnel or similar tests, without exact knowledge on the transfer function, exact order of the plant, etc. Cohen and Bossert[1] designed the following LTI controller:

$$C(s) = \frac{0.00842(s + 7.895)(s^2 + 0.108s + 0.3393)}{(s + 0.07895)(s^2 + 4s + 8)} \tag{34}$$

With this compensator, one obtains the open-loop Bode plot of Fig. 2 and the closed loop of Fig. 3. The step response of the plant in both situations is shown in Fig. 4.

Although the nominal response is satisfactory, the degraded situation shows a rather oscillatory time response. Unfortunately, adaptive control cannot be directly applied to this autopilot with any confidence because the plant is nonminimum phase and therefore not ASPR. However, as it was mentioned, if a plant is stabilizable (or maintains its stability) with some controller, augmenting the plant with the inverse of this controller as a PFC makes it minimum phase. Because we want the PFC to be strictly causal and of relative degree one, we are interested in using the inverse of a noncausal controller (or a noncausal part of controller) of relative degree 1.

Here, we complement Eq. (34) with a simple (fictitious) proportional–derivative (PD) controller that maintains plant stability. The selected PD controller is

$$C_2(s) = 1 + s/5 = (s + 5)/5 \tag{35}$$

We try to avoid the overburden of too many plots here, and yet one could easily see (from any standard frequency-domain plot) that the new (fictitiously) stabilized plant has a gain margin of

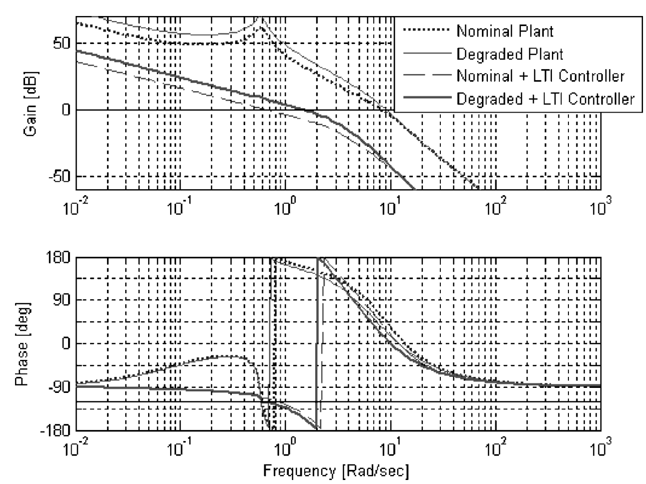

Fig. 2 Open-loop Bode plot of the plant with and without LTI controller.

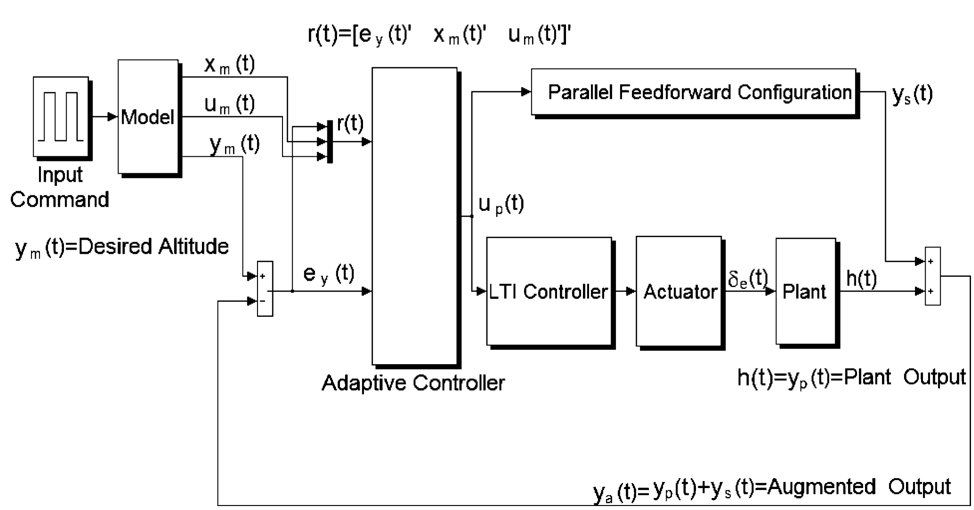

Fig. 1 Adaptive altitude hold block diagram.

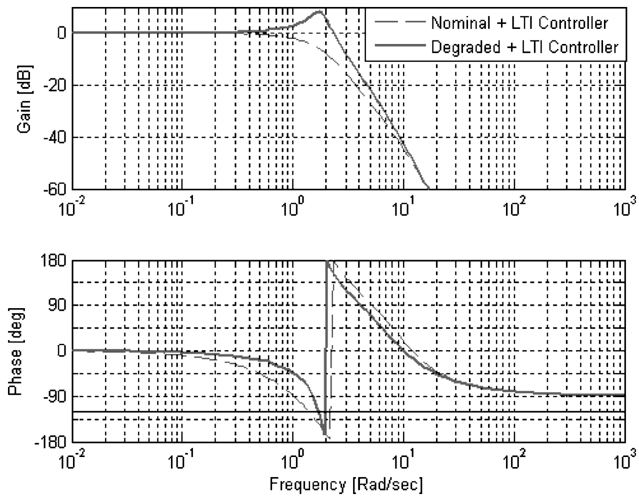

Fig. 3 Closed-loop Bode plot of the plant with LTI controller.

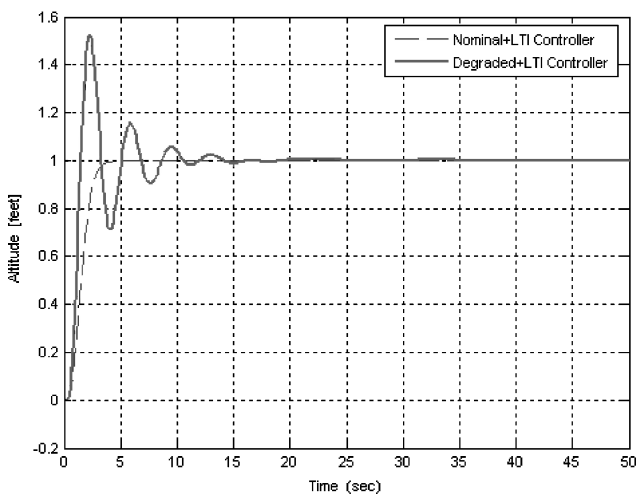

Fig. 4 Step response of plant with LTI controller.

1.4 (3 dB) in the degraded situation. Therefore, we implement the PFC configuration with a gain of $1/1.4 = 0.7$:

$$C_2^{-1}(s) = 0.7/(1 + s/5) = 3.5/(s + 5) \tag{36}$$

A root-locus plot would show that all zeros of the augmented system $G_a(s) = G(s) + C_2^{-1}(s)$ are located in the open left half-plane in both situations, namely, when $G(s) = G_1(s)$ and when $G(s) = G_2(s)$. Furthermore, because the relative degree of the augmented systems is one, as imposed by the PFC, both augmented systems are ASPR, and SAC can be applied with confidence.

The adaptive control system was simulated using MATLAB® and SIMULINK. The plant is required to follow the output of a simple first-order model with the desired plant behavior, having the transfer function representation

$$M(s) = 1/(1 + s/2) \tag{37}$$

The adaptation parameters are $\Gamma_I = \mathrm{diag}\{100 \;\; 10 \;\; 10\}$; $\Gamma_p = \mathrm{diag}\{0 \;\; 0 \;\; 100\}$; $\sigma = 0.001$.

Because the controlled variable of interest is the actual, rather than the augmented, plant output, we will show both the augmented output (the control variable of the adaptive controller) and the plant output (the actual control variable of the design).

As shown in Fig. 5, the augmented plant output closely follows the desired model output, both under the nominal and in the degraded conditions. This satisfactory performance of the augmented plant is shown only to confirm that SAC performs well when the ASPR conditions are satisfied. The designer, however, cannot be satisfied

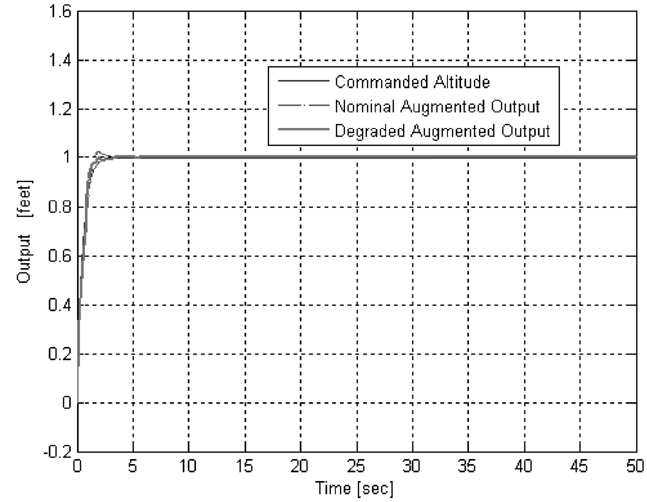

Fig. 5 Augmented plant output with LTI controller and SAC.

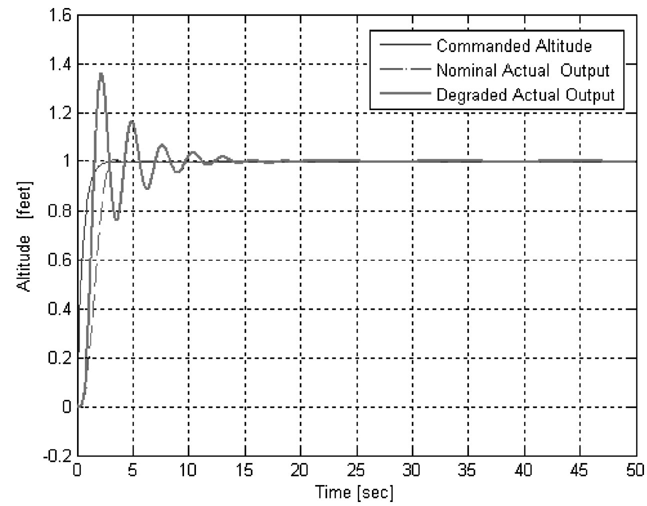

Fig. 6 Actual plant output with LTI controller and SAC.

with the behavior of the augmented plant because, as a result of the parallel feedforward configuration, one can expect to see that the plant output follows with some error.

The actual output response of the nominal plant is also sufficiently close to the desired model output to be called satisfactory (Fig. 6). We note, though, that the performance is somewhat better that the performance shown by all controllers used in Cohen and Bossert.[1] Furthermore, with the fulfillment of the ASPR conditions the stability with SAC is not only shown but also guaranteed.

However, we consider the performance under degraded conditions to be less than satisfactory. The discrepancy between the satisfactory performance of the augmented system and that of the original plant is caused by the large PFC gain compared with the plant gain. As the gain margin with PD is small, the gain of the PFC, its inverse, is too large. Because the designer motivation for using the simple adaptive control methodology is to improve upon the behavior of the fixed controller in changing environments, we consider that at this stage this application of SAC is not satisfactory.

VII. Control Design Revisited

As Cohen and Bossert mention,[1] the example of this paper is very difficult, and it was especially selected to illustrate the difficulty of design. The stationary controller designed by using the nominal plant parameters fails under deteriorated conditions. On the other hand, if one used the deteriorated conditions as the object of design, this would diminish the performance under nominal conditions. The fuzzy-logic controller[1] improves performance under deteriorated

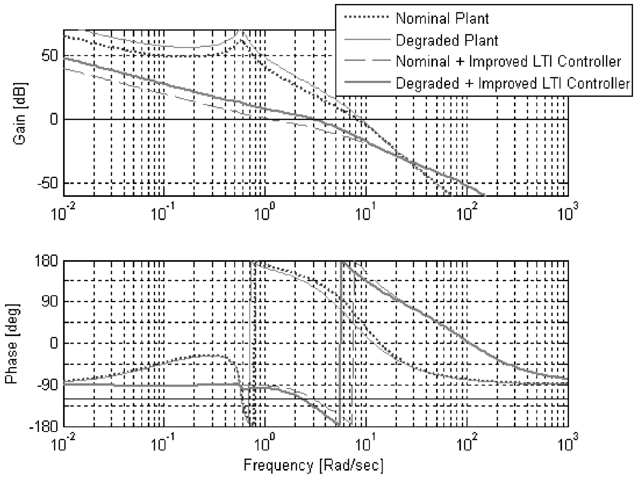

Fig. 7 Open-loop Bode plot of the plant with improved LTI controller.

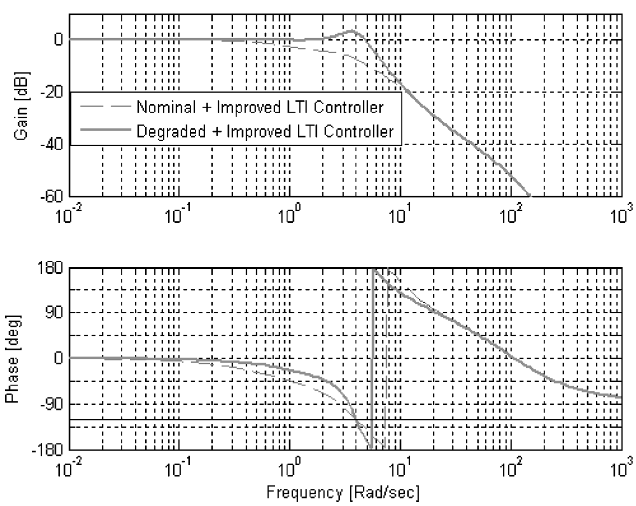

Fig. 8 Closed-loop Bode plot of the plant with improved LTI controller.

conditions, although its use seems to result in lower performance than the fixed controller under nominal conditions.

The basic classical controller design used by Cohen and Bossert[1] manages to transform a bad system into an almost acceptable one. However, before either going on with the attempts to improve the performance with modern controllers or just giving up we took a second look at the classical control design. Although the controller seemed to be optimal at first look, a careful second look at the Bode plots of the stabilized system (Figs. 2 and 3) might reveal that some more phase lead is needed at proper frequency range. To improve the shape of the frequency response, two new phase-lead links were added, and the gain was adjusted, resulting in the following improved LTI controller:

$$C_{\text{new}}(s) = 6.923\frac{(0.333s+1)^2}{(0.01s+1)^2}C(s)$$

$$= \frac{0.0126(s+7.895)(s^2+0.108s+0.3393)}{(s+0.07895)(s^2+4s+8)}\frac{(0.333s+1)^2}{(0.01s+1)^2} \tag{38}$$

Figures 7 and 8 show the open-loop and the closed-loop Bode plots of the plant with improved LTI controller, and Fig. 9 shows the improved step response under both conditions. The performance with the improved LTI controller is superior to all results presented in Cohen and Bossert.[1] Without trying to diminish the importance of adaptive control, fuzzy logic, or any other modern control design methodology, the results of this section can illustrate why experienced control design practitioners might stubbornly stick to the safety of classical LTI control design, even in most difficult uncertain environments.

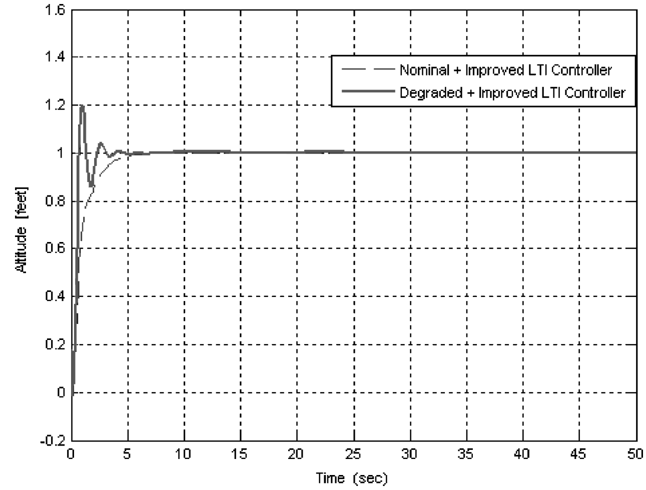

Fig. 9 Step response of the plant with improved LTI controller.

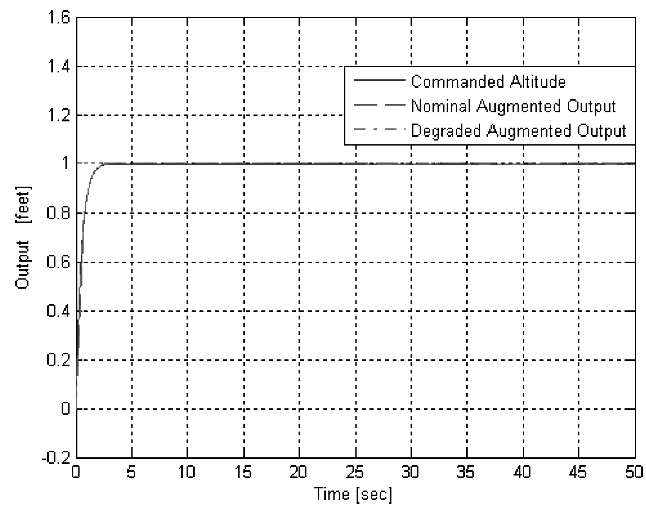

Fig. 10 Augmented plant output with improved LTI controller and SAC.

If the designer is satisfied with the resulting performance in both situations, the design could just end here, with a constant parameter controller. We assume, however, that one wants the plane to behave very closely to the ideal behavior in both cases. Therefore, we attempt to test whether simple adaptive control methods can improve upon stationary controllers, and we repeat here the SAC design techniques that we used in the preceding section. Inspecting the Bode plots of the system with the controller [Eq. (38)], we first add a fictitious PD link to the existent controller:

$$C_2(s) = 1 + s/5 = (s+5)/5 \tag{39}$$

It is easy to check that the system stabilized with fictitious PD controller shows a gain margin of more than 2.5. Yet, it is worth mentioning that the designer cannot actually implement Eq. (39) to improve the performance of the LTI controller because it would result in an improper transfer function. Instead, we implement the strictly proper PFC configuration

$$C_2^{-1}(s) = (1/2.5)[1/(1+s/5)] = 2/(s+5) \tag{40}$$

Both augmented systems are minimum phase and of relative degree one and are, therefore, ASPR. The augmented output behaves very well with SAC in both situations (Fig. 10), as expected. This time, however, the actual plant output (Fig. 11) also performs very well in both cases.

In both situations, the actual plant behavior is now very close to the desired ideal behavior represented by the model. One can see

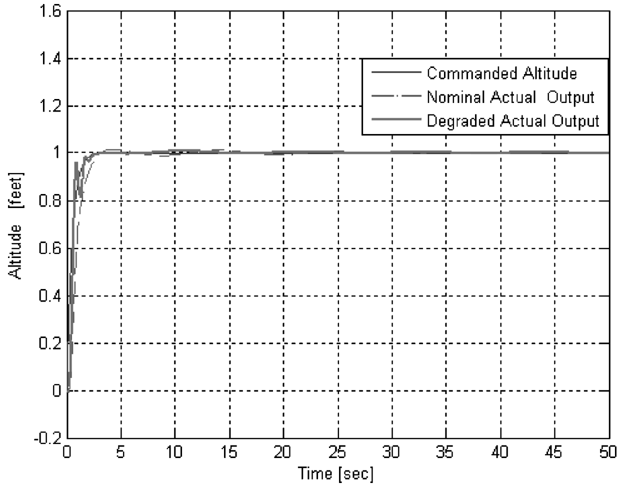

Fig. 11 Actual plant output with improved LTI controller and SAC.

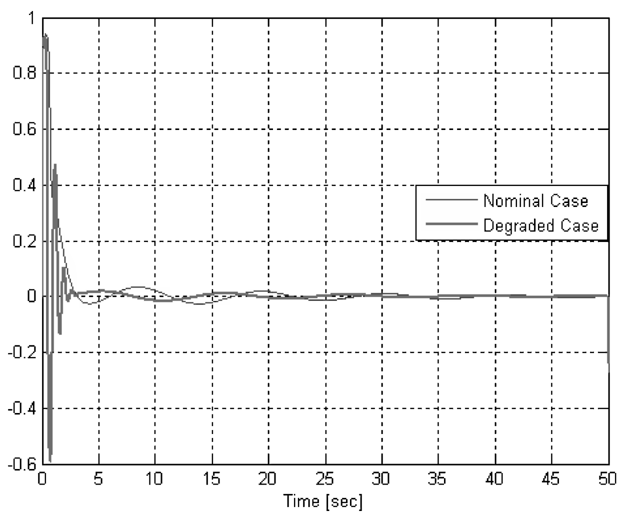

Fig. 12 Control signal u_p.

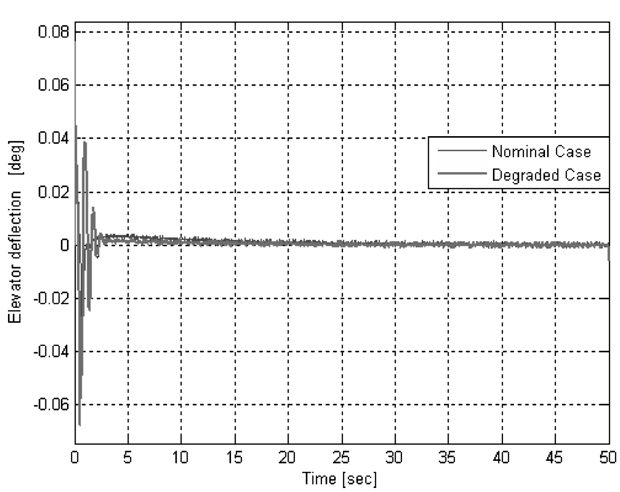

Fig. 13 Elevator response δ_e.

that, although the plant changes, the response remains similar in both cases. The particular application of this paper is very difficult, and the gain margin is not very high in any of the cases just studied. Still, the satisfactory result (Fig. 11) was obtained because of the lower PFC gain that was possible as a result of the larger gain margin of the fictitious plant with the improved LTI controller. The higher the gain margin of the linear system, the lower the PFC gain and the closer the actual output to the desired output.

This desired behavior is possible because the adaptive control gains are such that the resulting control signal (Fig. 12) can be different so that it can maintain small tracking errors in different situations. The high amplitudes shown in Fig. 12 represent only the amplitudes of the digital adaptive control signal. The elevator amplitude is represented in Fig. 13 and shows customary values.

The success of the application was offered only as an illustration. The stability of implementation is guaranteed and can suggest fulfilling the SPR conditions that have also been shown to be needed to guarantee stability with fuzzy controllers.[49]

VIII. Conclusions

This paper uses a difficult nonminimum phase example from the technical literature to test the advantage of using a combination of classical and simple adaptive control design in a realistic environment with uncertainty. Preliminary conditions, which can be created based on the prior knowledge on stabilizability of the controlled plant, guarantee stability with nonstationary adaptive controllers. The paper shows that direct application of adaptive control without good preliminary understanding and use of the prior knowledge on the plant can still result in failure as far as performance is concerned. However, when prior knowledge is appropriately used, being able to supply the proper gains and control signal for the proper situation, simple adaptive control can allow reaching the performance that remains very close to the ideal behavior under all expected conditions.

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